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LIMITING INTENSIFICATION OF HEAT EXCHANGE IN TUBES DUE TO ARTIFICIAL TURBULIZATION OF THE FLOW

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A theoretical model for calculating the limiting isothermal parameters of heat exchange and resistance under the conditions of intensification of heat exchange in tubes due to turbulization of the flow has been developed. The model describes the corresponding processes for a wide range of Reynolds and Prandtl numbers, which enables one to predict the limits of intensification of heat exchange with a much higher degree of accuracy than the existing methods.

Introduction. Intensification of convective heat exchange due to artificial turbulization of the flow is one basic practical method for improving the efficiency of heat-exchange apparatuses. In this investigation, by the intensification of heat exchange we mean turbulizers periodically arranged on the tube surface and ensuring periodic separations and attachments of the flow.

To select the parameters of intensification of heat exchange one must know its limits. By comparing its values to the results obtained in actual practice one can evaluate possible limits of intensification of heat exchange by this method.

We consider the following formulation of the problem: a turbulized flow is modeled by a three-layer scheme. Then we consider the conditions under which its limiting turbulization is realized. Next we calculate the values of the hydraulic resistance and the heat exchange under the assumption that the flow is turbulized to such a state in which each component of the thermal resistance is minimum. The limiting hydraulic resistance is calculated within the framework of this investigation precisely on the basis of the absolute occupation of all the sublayers of the turbulized flow. In other words, in the case of limiting intensification each of the sublayers is turbulized in the limit, which is possible only for certain relations between the geometric characteristics of turbulizers and the regime of flow (this has been noted, for example, in [1]).

Consequently, it is necessary to develop a theoretical model within the framework of which it would be possible to calculate the limiting heat exchange from the results of calculation of the resistance.

Modeling of Intensification of Heat Exchange. The problem on limiting heat exchange was solved for the first time in [2, 3], where consideration was given to a rather rough two-layer scheme. Clearly, a virtually viscous sublayer cannot pass to the region of the turbulent core at once, abruptly: there must exist a transition region with intermediate passages to both the sublayer and the turbulent core.

It is well known [1, 3, 4] that thermal resistance is nonuniformly distributed over the cross section of a turbulent flow in a smooth tube; it is basically concentrated in a narrow wall region; the viscous sublayer and the intermediate (buffer) region in the case of gas flow account for 85% of the temperature head in the turbulent flow. Consequently, one must turbulize precisely this region; additional turbulization of the turbulent core has no meaning since it leads to a growth in the hydraulic resistance without increasing the heat transfer.

The maximum heat exchange in tubes with transverse steps is observed in the case where the distance between the steps is 5 to 12 times larger than the turbulizer height [5–7].

Thus, in the case of the limiting intensification of heat exchange in tubes due to turbulization of the flow each component of the thermal resistance will be in the limiting state, namely: the value of the viscous sublayer remains constant in any external turbulization; the intermediate region (on the average) cannot be larger than $h/2$, in practice; the flow core cannot be turbulized to a larger extent than in jet flow.

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Calculation of the Limiting Isothermal Resistance. Now we should determine the resistance coefficient of the limiting turbulized flow. In [1] it has been proved that in the case of the limiting turbulization the velocity profile for the turbulent flow core is expressed by the following relation:

$$\frac{w_x}{\overline{w}_x} = \frac{\sqrt{\xi}}{2\sqrt{2}} B + \frac{\xi}{0.416} \left[\left(1 - \frac{4A\sqrt{2}}{\sqrt{\xi}\,\mathrm{Re}} \right)^2 - \frac{R^2}{2} \right]. \tag{1}$$

We have A = 15 and B = 6.5 for Re = 10^4 and A = 114 and B = 8.5 for Re = 10^5.

Let us determine the average velocity of the turbulized flow:

$$\overline{w}_x = 2\int_0^1 w_x R dR = 2 \left\langle \int_{1-\frac{4A\sqrt{2}}{\sqrt{\xi}\,\mathrm{Re}}}^{1} \overline{w}_x \frac{\kappa\,\mathrm{Re}\,\xi}{16} (1-R)\, RdR + \right.$$

$$\left. + \int_0^{1-\frac{4A\sqrt{2}}{\sqrt{\xi}\,\mathrm{Re}}} \overline{w}_x \left\{ \frac{\sqrt{\xi}}{2\sqrt{2}} B + \frac{\xi}{0.416} \left[\left(1 - \frac{4A\sqrt{2}}{\sqrt{\xi}\,\mathrm{Re}} \right)^2 - \frac{R^2}{2} \right] \right\} RdR \right\rangle , \tag{2}$$

where κ = 0.443 [1].

The value of the resistance coefficient ξ of the turbulized flow is found upon integration of expression (2), subtraction by $\overline{w}_x$, and carrying out a number of trivial simplifications:

$$624\,\xi\,\mathrm{Re}^4 = 12\,\mathrm{Re}^2 \left(\xi\sqrt{\xi}\,\mathrm{Re}^2 - 8A\sqrt{2}\,\mathrm{Re}\,\xi + 32A^2\sqrt{\xi} \right) \left[13\sqrt{2}\,B + 125\sqrt{\xi} \left(1 - \frac{4A\sqrt{2}}{\sqrt{\xi}\,\mathrm{Re}} \right)^2 \right] +$$

$$+ 375 \left(-\xi^2\,\mathrm{Re}^4 + 16A\sqrt{2}\,\xi\sqrt{\xi}\,\mathrm{Re}^3 - 192A^2\xi\,\mathrm{Re}^2 + 512A^3\sqrt{2}\sqrt{\xi}\,\mathrm{Re} - 1024A^4 \right) +$$

$$+ 416A^2\kappa\sqrt{\xi}\,\mathrm{Re}^2 \left(3\sqrt{\xi}\,\mathrm{Re} - 8A\sqrt{2} \right). \tag{3}$$

Expression (3) is the exact equation for ξ, while the equation obtained in [1] is an approximate equation since integration in [1] has been carried out in dimensional form with a number of simplifications. Integration in dimensionless form (Eq. (2)) enabled us to totally eliminate this drawback, since in the equation obtained in [1] the dependence of the value of ξ on the tube radius R_0 is physically absurd.

It is obvious that Eq. (3) can be solved only numerically.

We particularly emphasize that the value of the resistance coefficient obtained using Eq. (3) determines the friction resistance, i.e., the minimum value of the resistance coefficient [1]. Determination of the pressure resistance is beyond the scope of this investigation; it is not taken into account by this model of resistance and heat exchange. The procedure of calculation of all the components of the hydraulic resistance has been considered in detail in [3, 4, 7].

Calculation of the Limiting Isothermal Heat Exchange. Heat-exchange characteristics are determined on the basis of the Lyon integral [8]. In the first approximation, to calculate the limiting heat exchange we employed the Lyon integral for $w_x / \overline{w}_x \cong 1$ similarly to the approach used in [1]:

$$\mathrm{Nu} = \frac{2}{\displaystyle\int_0^1 \frac{R^3}{1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t} \frac{\mu_t}{\mu}} dR} =$$

TABLE 1. Calculated Values of the Limiting Isothermal Resistance and Parameters of Heat Exchange for Re = 10^4 and Re = 10^5

Parameters	Data of [2]		Nonlinear equation (2)		Formula (4)		Formula (5)	
Re	10^4	10^5	10^4	10^5	10^4	10^5	10^4	10^5
ξ	0.123	0.084	0.125	0.07	–	–	–	–
ξ/ξ_0	3.89	4.67	3.96	4.05	–	–	–	–
Nu	128.0	723.3	–	–	129.3	676.1	128.9	681.0
$\mathrm{Nu}/\mathrm{Nu}_0$	4.06	3.62	–	–	4.09	3.39	4.08	3.41
Ξ	1.044	0.845	–	–	1.033	0.836	1.029	0.843

$$= \frac{2}{\displaystyle\int_0^{1-\frac{A}{\mathrm{Re}\sqrt{\xi/32}}} \frac{R^3}{1+\frac{\mathrm{Pr}}{\mathrm{Pr_t}}\sigma\,\mathrm{Re}}\,dR + \int_{1-\frac{A}{\mathrm{Re}\sqrt{\xi/32}}}^{1-\frac{6}{\mathrm{Re}\sqrt{\xi/32}}} \frac{R^3}{1+\frac{\mathrm{Pr}}{\mathrm{Pr_t}}\left(\frac{\mu_t}{\mu}\right)_{\mathrm{tr.r}}}\,dR + \int_{1-\frac{6}{\mathrm{Re}\sqrt{\xi/32}}}^{1} dR}\,, \tag{4}$$

where $\mathrm{Pr_t} = 0.7$ and $\sigma = 0.013$ is the constant characterizing the initial turbulence [1].

Upon integration of expression (4) we obtain

$$\mathrm{Nu} = 2 \Bigg/ \left\{ \frac{\left(1-\frac{A}{\mathrm{Re}\sqrt{\xi/32}}\right)^4}{4\frac{\mathrm{Pr}}{\mathrm{Pr_t}}\sigma\,\mathrm{Re}} + \frac{\left(1-\frac{6}{\mathrm{Re}\sqrt{\xi/32}}\right)^4 - \left(1-\frac{A}{\mathrm{Re}\sqrt{\xi/32}}\right)^4}{4\left(1+\frac{\mathrm{Pr}}{\mathrm{Pr_t}}\frac{A}{B}\right)} + \frac{6}{\mathrm{Re}\sqrt{\frac{\xi}{32}}} \right\}. \tag{5}$$

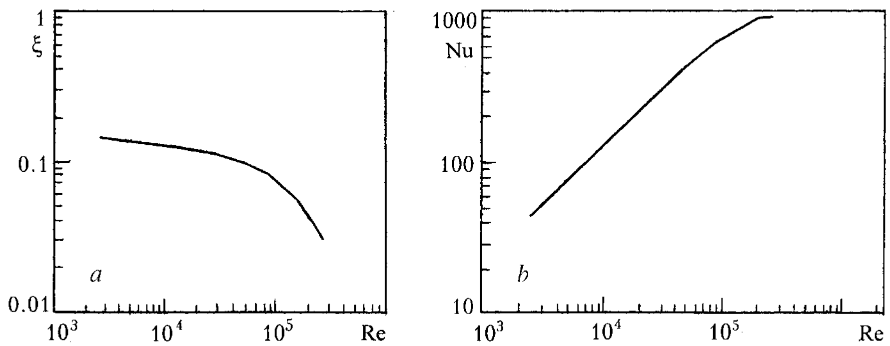
It is quite natural that calculation of the limiting heat exchange (4) on the basis of the Lyon integral for $w_x/\overline{w}_x \cong 1$ is approximate to an extent; therefore, it makes sense to evaluate it on the basis of the Lyon integral without additional assumptions [8]:

$$\mathrm{Nu} = 1 \Bigg/ \left[2\int_0^1 \frac{\left(\int_0^R \frac{w_x}{\overline{w}_x} R\,dR\right)^2}{\left(1+\frac{\mathrm{Pr}}{\mathrm{Pr_t}}\frac{\mu_t}{\mu}\right)R}\,dR \right]. \tag{6}$$

It is most rational to compute integral (6) numerically.

The values of the limiting isothermal parameters of heat exchange and resistance in turbulent channel flow due to turbulization of the flow calculated on the basis of formulas (3), (5), and (6) for Pr = 0.7 are given in Table 1 for Re = 10^4 and Re = 10^5; for the sake of comparison we also give the corresponding calculated data from [1].

As is obvious from the table, for all cases ξ decreases with increase in the Reynolds number but the ratio ξ/ξ_0 somewhat increases. When Re = $5\cdot10^4$ the ratio $(\mathrm{Nu}/\mathrm{Nu}_0)/(\xi/\xi_0)$ is more than unity, which points to a violation of the Reynolds analogy in favor of the transfer of heat. The above circumstance is attributed to the influence of jet flow for which $\mathrm{Pr_t} = 0.7$. As the Reynolds number increases, the effect of the prevailing growth in the intensity of heat transfer compared to the increase in the momentum loss disappears and when Re = 10^5 the intensity of momentum transfer dominates that of heat transfer.



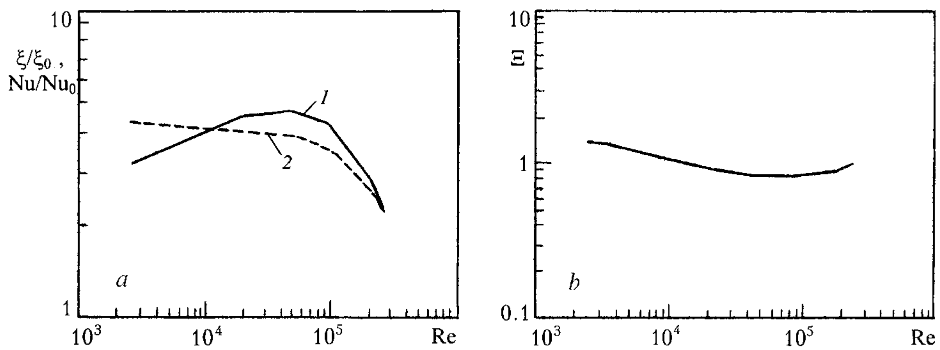


Fig. 1. Limiting resistance and heat exchange for air in turbulent channel flow vs. Reynolds number.

Fig. 2. Limiting relative heat exchange and resistance and limiting characteristic Ξ in turbulent channel flow vs. Reynolds number: 1) ξ/ξ_0; 2) Nu/Nu_0 (Dittus and Boelter).

The above regularities are in good qualitative agreement with experimental data for tubes with turbulizers which demonstrate that the transition region has an advantage from the viewpoint of intensification of heat exchange [3–6], since the maximum of the parameter $(Nu/Nu_0)/(\xi/\xi_0)$ falls approximately on $Re = 10^4$, after which $(Nu/Nu_0)/(\xi/\xi_0)$ decreases.

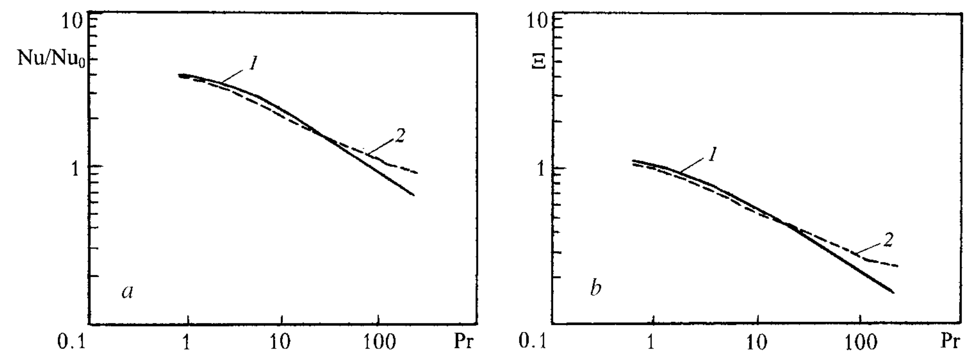
The calculated data on the limiting isothermal heat exchange and resistance in turbulent channel flow due to turbulization of the flow which are given in the table point to the fact that the exact solutions of the problem on limiting heat exchange and resistance (3), (5), and (6) obtained within the framework of this investigation differ appreciably from the data of [1], particularly in the region of $Re = 10^5$. Consequently, in calculating the limiting isothermal parameters of heat exchange and resistance in the case of turbulent channel flow due to turbulization of the flow, one must employ precisely this procedure: it is more accurate than the procedure of [1].

It is obvious from the table that the calculated data on isothermal heat exchange in turbulent channel flow due to turbulization of the flow which have been obtained using formulas (5) and (6) differ little (by less than 1%); this demonstrates a sufficient accuracy of the theoretical model developed for finding the limiting isothermal values of the heat exchange and the resistance.

Figure 1a gives the calculated values of the limiting coefficient of resistance as a function of the Reynolds number which have been obtained by numerical solution of Eq. (3). It is obvious that the resistance coefficient decreases with increase in the Reynolds number.

The value of the limiting heat exchange as a function of the Reynolds number obtained using formulas (5) and (6) is given in Fig. 1b, from which it is clear that the Nusselt number increases insignificantly after $Re = 10^5$.

Figure 2a shows the values of ξ/ξ_0 and Nu/Nu_0 as functions of the Reynolds number; they point to the fact that in the region of $Re \approx 10^4$ heat transfer dominates momentum transfer; subsequently, up to $Re \approx 5\cdot10^4$, heat transfer becomes lower than momentum transfer, after which the difference between them decreases. The above regularity



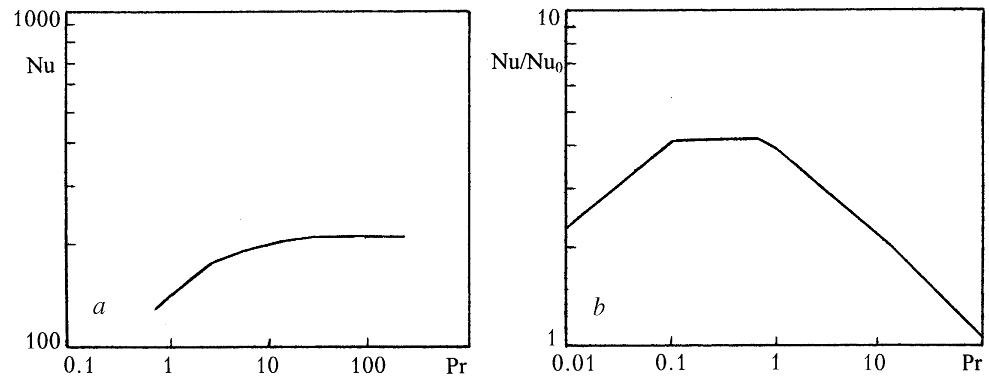


Fig. 3. Limiting heat exchange and characteristic Ξ in turbulent channel flow vs. Prandtl number for Re = 10,000: 1) Nu/Nu_0 (Dittus and Boelter); 2) Nu/Nu_0 (Petukhov).

Fig. 4. Absolute value of the limiting heat exchange vs. Prandtl number in a wide range of the latter in turbulent channel flow for Re = 10,000.

is reflected in Fig. 2b, where the dependence $\Xi(\mathrm{Re}) = : \dfrac{Nu/Nu_0}{\xi/\xi_0}(\mathrm{Re})$ is shown. Its feature of interest is the presence of the minimum of the function Ξ(Re) for $\mathrm{Re} \approx 5 \cdot 10^4$, after which the value of Ξ increases with Reynolds number, reaching unity for $\mathrm{Re} \approx 2.5 \cdot 10^4$. We emphasize that before the minimum is reached the value of Ξ becomes equal to unity for $\mathrm{Re} = 1.17 \cdot 10^4$. The character of the dependence shown in Fig. 2b also demonstrates that the most suitable for intensification is the transition region, which has already been noted.

The procedure developed for calculation of the limiting isothermal parameters of heat exchange and resistance in turbulent channel flow due to turbulization of the flow enables us to determine them not only as functions of the Reynolds number but as functions as the Prandtl number as well. Figure 3a gives the ratio $\dfrac{Nu}{Nu_0}(\mathrm{Pr})$ (Nu_0 is the Nusselt number for a smooth tube; it has been calculated from both the Dittus–Boelter (Boulter) formula and the Petukhov formula [8], which better corresponds to experimental data for large Prandtl numbers) for $\mathrm{Re} = 10^4$. It is seen that as the Prandtl number increases we have a decrease in the limiting relative heat exchange; this is in complete agreement with the physical principles of heat-exchange processes realized under the conditions of limiting intensification due to turbulization of the flow. The dependence of the characteristic Ξ on the Prandtl number given in Fig. 3b for $\mathrm{Re} = 10^4$ also confirms that the limiting intensification of heat exchange due to turbulization of the flow for heat-transfer agents in the form of dropping liquids is less effective than for gas heat-transfer agents. The absolute values of the limiting Nusselt number as a function of the Prandtl number for $\mathrm{Re} = 10^4$ are shown in Fig. 4a; they point to the fact that the Prandtl number approaching $\mathrm{Pr} \approx 100$ exerts no influence on the absolute values of the limiting Nusselt numbers, which remain constant, in practice. The above theoretical regularity totally confirms the experimental data demonstrating that the limiting intensification of heat exchange due to turbulization of the flow with heat-transfer

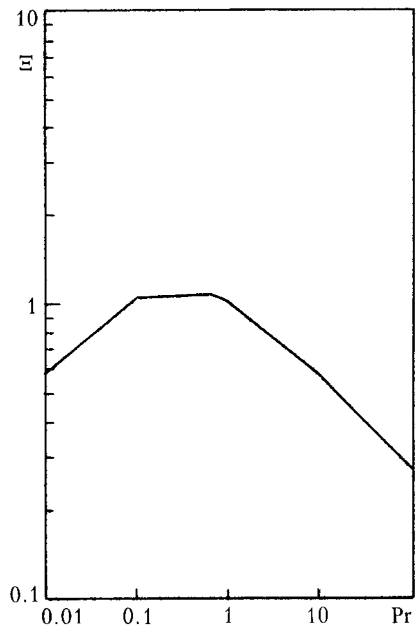


Fig. 5. Limiting characteristic Ξ vs. Prandtl number in a wide range of the latter in turbulent channel flow for Re = 10,000.

agents in the form of dropping liquids (relatively large Prandtl numbers) reaches "saturation" [5, 6], which is not observed for gas heat-transfer agents [5, 6]. We emphasize that the above "saturation" effect can also be observed for gas heat-transfer agents but only in the case of higher steps [5, 6]. This case is not considered in this investigation, since here the limiting occupation of all the thermal sublayers will no longer occur.

Of prime importance is the problem with what heat-transfer agents can one attain the maximum effect from the viewpoint of intensification of heat transfer: with liquid metals, gases, or dropping liquids.

Figure 4b and Fig. 5 give the dependences $\frac{Nu}{Nu_0}(Pr)$ and $\frac{Nu/Nu_0}{\xi/\xi_0}(Pr)$ for $Re = 10^4$ (the values of Nu_0 have been calculated from formulas most adequately corresponding to experimental data for the appropriate ranges of Prandtl numbers: we employed the Petukhov formula for large Prandtl numbers, the Dittus–Boelter formula for moderate numbers, and the Subbotin formula [8] for small Prandtl numbers) respectively for a very wide range of Prandtl numbers: Pr = 0.01–100. It is seen that the maxima of the ratios $\frac{Nu}{Nu_0}(Pr)$ and $\frac{Nu/Nu_0}{\xi/\xi_0}(Pr)$ are located at $Pr \approx 10^{-1}$–1. Consequently, purely theoretically we have obtained one more piece of decisive evidence that intensification of heat exchange due to turbulization of the flow is more preferable for gas heat-transfer agents than for liquid metals and dropping liquids.

CONCLUSIONS

In this investigation, the problem of calculation of the limiting isothermal values of the heat exchange and the resistance in turbulent channel flow due to turbulization of the flow has been solved theoretically and the corresponding drawbacks occurring in the analogous previous works have been eliminated. The calculation results on the limiting heat exchange and resistance have been obtained for a wide range of Reynolds and Prandtl numbers. It has been proved theoretically that intensification of heat exchange through turbulization of the flow is more preferable for gases than for liquid metals and dropping liquids. The method developed in this investigation enables one to predict the limits of intensification of heat exchange with a higher degree of accuracy.

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NOTATION

x and y, longitudinal and radial coordinates reckoned from the wall, m; D, inside diameter of the tube, m; $R_0 = D/2$, internal radius of the tube, m; $R = 1 - 2y/D$, relative radius of the tube; A, B, κ, and σ, constants; Nu, Nusselt number; Re, Reynolds number; Pr, Prandtl number; $\mathrm{Pr_t}$, turbulent Prandtl number; h, turbulizer height, m; w_x, axial velocity component, m/sec; $\overline{w}_x$, mean-flow-rate velocity, m/sec; ξ_0, resistance coefficient for a smooth tube according to Blasius; $\mathrm{Nu_0}$, Nusselt number for a smooth tube according to Dittus and Boelter; μ and μ_t, dynamic and turbulent viscosities respectively, Pa·sec; $(\mu_t/\mu)_{\mathrm{tr.r}}$, turbulent-to-dynamic viscosity ratio for the transition (buffer) region. Subscripts: t, turbulent; tr.r, transition region; 0, values for a smooth tube.

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